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SUM-FREQUENCY GENERATION FROM PHOTON NUMBER SQUEEZED LIGHT

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Abstract

We investigate the quantum fluctuations of the fields produced in sum-frequency (SF) generation from light initially in the photon number squeezed state. It is found that, to the fourth power term, the output SF light is sub-Poissonian whereas the quantum fluctuations of the input beams increase. Quantum anticorrelation also exists in SF generation.

1 Introduction

In recent years squeezed states of light have been successfully produced in several laboratories and certain applications also demonstrated.[1] In future applications it may be desirable to change the frequency of a light beam already in the squeezed state, for example by frequency doubling, parametric down conversion or four wave mixing. Three wave processes are preferable since the second order nonlinear susceptibility is much larger than the third order susceptibility. Second harmonic generation from a pump beam initially in a squeezed state has been discussed[2,3] as well as SF/DF generation and degenerate parametric down conversion of quadrature squeezed light.[3,4] In this paper we shall consider SF generation from input fields initially in the photon number squeezed or sub-Poissonian state.

2 Sum-frequency generation

As shown in Fig. 1 we take both input fields, of frequency ω_1 and ω_2, and the sum-frequency ω_3 field ($\omega_3 = \omega_2 + \omega_1$) to be in a collinear geometry. The input beams are incident on a nonlinear medium such as a crystal with an effective second order nonlinear coefficient χ'. The field Hamiltonian for ideal SF generation ignoring losses is

$$H = \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar\omega_3 a_3^\dagger a_3 + \hbar\chi'(a_1 a_2 a_3^\dagger + \text{h.c.}) \tag{1}$$

where a_1, a_2 and a_3 are the annihilation operators for the two input beams and the resultant SF beam, respectively.

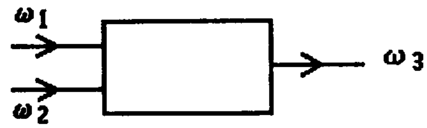

FIG. 1 Sum-frequency generation in a nonlinear crystal.

We introduce the slowly varying operators

$$c_j(t) = a_j\, e^{i\omega_j t} \tag{2}$$

where the subscript $j = 1, 2$ or 3 refers to the two input or SF waves, respectively, as throughout our paper. For propagation along the z-axis at a velocity u in a lossless medium we make the conversion $z = ut$. Since $\omega_3 = \omega_1 + \omega_2$ and substituting $\chi = \chi'/u$ we deduce the equations of motion for the slowly varying operators from Eq. (1) to be

$$\begin{aligned} \frac{dc_1(z)}{dz} &= -i\chi c_2^\dagger(z)c_3(z), \\ \frac{dc_2(z)}{dz} &= -i\chi c_1^\dagger(z)c_3(z), \\ \frac{dc_3(z)}{dz} &= -i\chi c_1(z)c_2(z). \end{aligned} \tag{3}$$

In the short path approximation for $\chi z \ll 1$ we assume the solution of Eq. (3) to take the form

$$c_j(z) = c_j(0) + (\chi z)B_j + (\chi z)^2 C_j + (\chi z)^3 D_j + (\chi z)^4 E_j + \cdots \tag{4}$$

where $c_j(0)$ denotes the incident fundamental field at $t = 0$. It may be seen that $c_j(z)$ still satisfies the commutation relation

$$[c_i(z), c_j^\dagger(z)] = \delta_{ij}. \tag{5}$$

Substituting into Eq. (3) we obtain the solutions for $c_1(z)$, $c_2(z)$ and $c_3(z)$.

To discuss the photon number fluctuations of the quantized fields we consider the variance $\langle \Delta n_j^2(z)\rangle$ or the Fano factor

$$\sigma_j = \frac{\langle \Delta n_j^2(z)\rangle}{\langle n_j(z)\rangle} \tag{6}$$

where $n_j(z) = c_j^\dagger(z)c_j(z)$, and $\langle \Delta n_j^2(z)\rangle \equiv \langle n_j^2(z)\rangle - \langle n_j(z)\rangle^2$. To obtain the above expressions we need to solve for the factors $c_k^\dagger c_k$, $c_k^{\dagger 2} c_k^2$ and $c_k^{\dagger 3} c_k^3$, where $c_k = c_k(0)$ and $k = 1$ or 2, then find their expectation values. If the input fields are in a photon number squeezed state we adopt the formalism given by Kitagawa and Yamamoto[5], that is,

$$c_k = e^{i\gamma_k n_k} a_k + \xi_k, \qquad \xi_k = -i\eta_k \alpha_k e^{i\phi_k}. \tag{7}$$

Here $n_k = a_k^\dagger a_k$ is the number operator and γ_k, η_k are constants chosen to maximise the squeezing for a given input field intensity proportional to $|\alpha_k|^2$. Using Eqs. (7) and after some tedious calculations we may obtain the expectation values $\langle c_k^\dagger c_k\rangle$ and higher terms with respect to the eigenstates of a_k, noting that $c_3(0)$ is a vacuum field. The Fano factors may thus be computed numerically for explicit values of α_k, γ_k and η_k. It is evident that the expressions for the Fano factors of the two input beams are identical if the subscripts $k = 1$ and 2 are interchanged.

Another quantity that reflects the quantum fluctuations of the light fields is the quantum correlation between the sum of the input photon numbers, $n_1(z) + n_2(z)$, and the SF photon number $n_3(z)$. That is, if we define $\langle n(z)\rangle = \langle n_1(z) + n_2(z) + n_3(z)\rangle$, then

$$\langle \Delta n^2(z)\rangle = \langle \Delta(n_1 + n_2)^2(z)\rangle + \langle \Delta n_3^2(z)\rangle + 2[\langle (n_1 + n_2)(z) n_3(z)\rangle - \langle (n_1 + n_2)(z)\rangle\langle n_3(z)\rangle], \quad (8)$$

and we define the Fano factor for the total sum of the photon numbers as

$$\sigma(z) = \frac{\langle \Delta n^2(z)\rangle}{\langle n(z)\rangle}. \quad (9)$$

The Fano factors for different pairs of the two input beam intensities are plotted against the effective interaction length χz in Figs. 2 to 4. It can be seen that σ_3 becomes less than 1 in all three cases, that is, the SF produced becomes sub-Poissonian. Moeover, as the interaction length increases the photon number fluctuations are reduced even further but at the expense of the input fields.

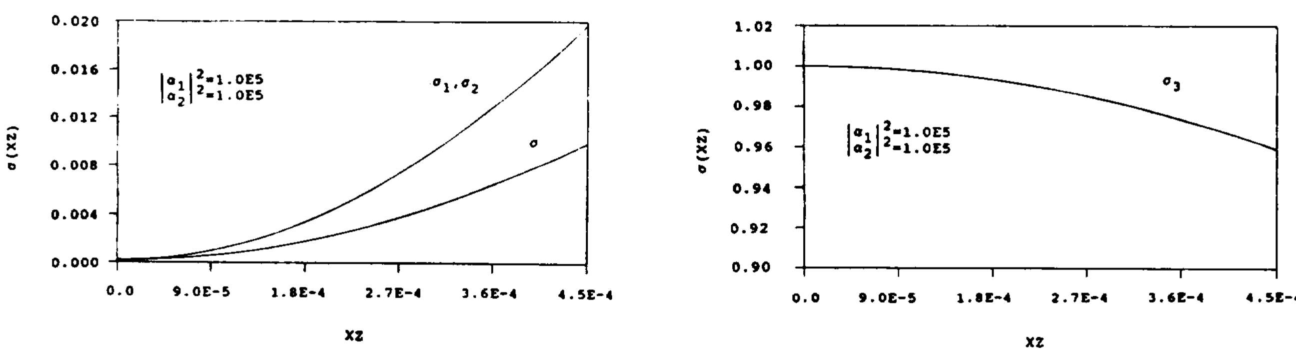

FIG. 2 Fano factors of the input and SF beams versus effective interaction length for sub-Poissonian inputs of equal intensity. Subscripts 1,2 and 3 denote the two inputs and the SF beam, respectively. σ is the Fano factor for the total photon number of the three beams.

As shown in Fig. 2, when the two input beams are of the same intensity, that is, $|\alpha_1|^2 = |\alpha_2|^2$ and assuming that their initial statistical properties are identical, then σ_1 and σ_2 are always the same, as may be expected. However, when one frequency e.g. ω_2 has a greater number of photons than the other, its Fano factor is less affected (see Fig. 3) since there is a greater number of unconverted photons left. The Fano factor σ for the sum of all three intensities also increases with interaction length, but it is always better i.e. less than that for the individual beams. This can be understood from the correlation between the three beams, since the creation of one SF photon is always associated with the annihilation of two input photons, one from each input beam.

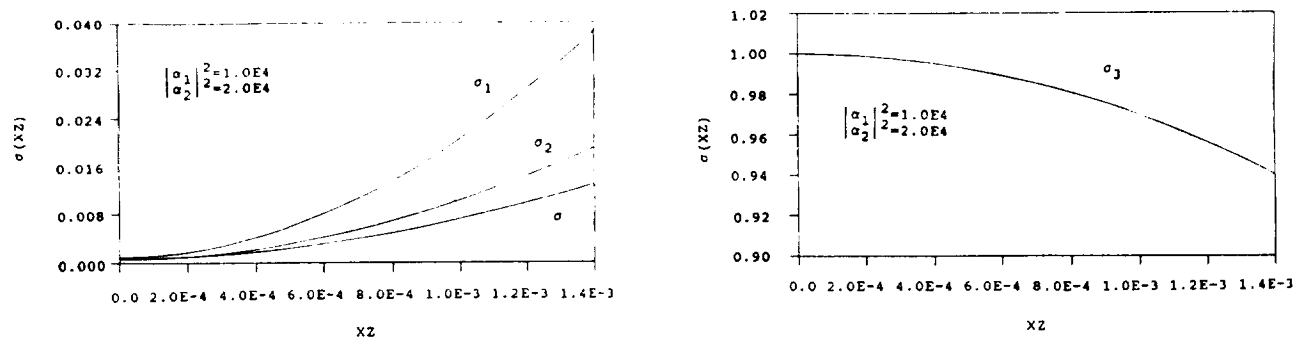

FIG. 3 Fano factors of the input and SF beams versus effective interaction length for sub-Poissonian inputs of different intensities. Subscripts 1,2 and 3 denote the two inputs and the SF beam, respectively. σ is the Fano factor for the total photon number of the three beams.

In Fig. 4 we have taken one input beam ω_2 to be sub-Poissonian whilst the other ω_1 is in a coherent state, with both having the same intensity. It can be seen that the curves of σ_2 and σ_3 behave similarly to those in the previous figure, but σ_1 remains constant at 1. This implies that the coherent input beam remains coherent throughout the interaction, with only the originally squeezed beam suffering increased quantum fluctuations. The physical reason for this is yet to be fully understood.

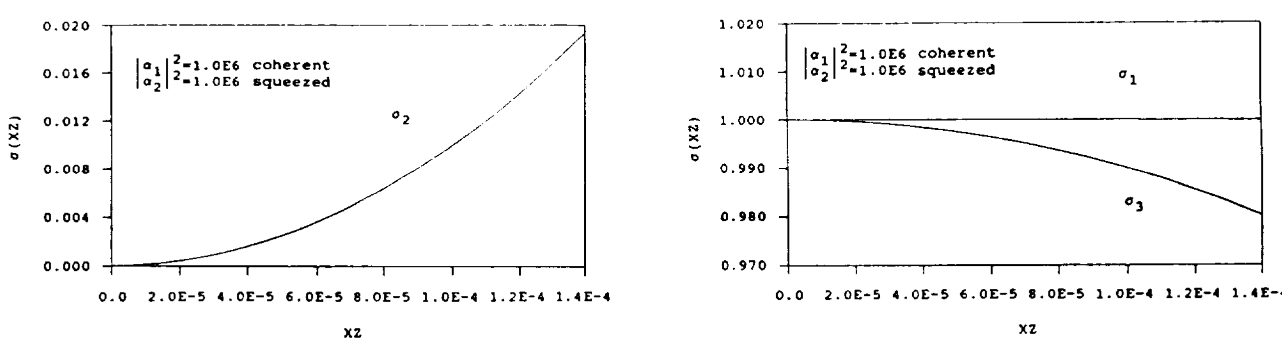

FIG. 4 Fano factors of the input and SF beams versus effective interaction length for inputs of equal intensities but different statistical nature. Subscripts 1,2 and 3 denote the two inputs and the DF beam, with ω_1 in a coherent state and ω_2 in a sub-Poissonian state. σ is the Fano factor for the total photon number of the three beams.

3 Conclusion

We have investigated the photon number fluctuations in SF generation with sub-Poissonian light as input and small effective interaction length. We find that the generated SF beam becomes sub-Poissonian and the quantum fluctuations decrease as the effective interaction length increases, but at the expense of the two input beams which become noisier. The Fano factor of the sum of the intensities of the three beams increases also with interaction length, but it will always be less than that of the other beams. This can be understood on the grounds of the quantum correlation between the input and SF fields. When one of the input beams is coherent and the other sub-Poissonian, the coherent beam remains coherent throughout the interaction, whilst the other becomes noisier. The generated SF beam, however, still becomes sub-Poissonian.

4 Acknowledgments

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